

Name: _____

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Key

1) Find impulse response of a system described by the following linear constant coefficient differential equation:

$$\frac{d^2 y}{dt^2} + 4 \frac{dy}{dt} + 3y(t) = 2 \frac{dx}{dt} + 8x(t)$$

$$\lambda^2 + 4\lambda + 3 = 0$$

$$(\lambda + 3)(\lambda + 1) = 0 \Rightarrow \lambda_1 = -3, \lambda_2 = -1$$

$$\text{So, } y_{zi} = c_1 e^{-t} + c_2 e^{-3t}$$

$$\text{f.c. } y_0(0) = 0, y'_0(0) = 1$$

$$c_1 + c_2 = 0$$

$$y'_0 = -c_1 e^{-t} - 3c_2 e^{-3t}$$

$$y'_0(0) = -c_1 - 3c_2 = 1$$

$$-c_1 + 3c_2 = 1$$

$$2c_2 = 1 \Rightarrow c_2 = \frac{1}{2}$$

$$c_1 = -\frac{1}{2}$$

$$h(t) = \mathcal{L}^{-1} \{ (2D + 8) \mathcal{L}^{-1} \{ \mathcal{L} \{ u(t) \} \} \}$$

$$= (2D + 8) \left(\frac{1}{2} e^{-t} - \frac{1}{2} e^{-3t} \right)$$

$$= \left(-e^{-t} + 3e^{-3t} + 4e^{-t} - 4e^{-3t} \right) u(t)$$

$$= \left(3e^{-t} - e^{-3t} \right) u(t)$$

2) The impulse response, $h(t)$, of an LTI system is shown in figure below. If this system is excited with an input signal $x(t)$, as shown in the figure, find its output $y(t)$?

Graphically

$$t < -1 \Rightarrow y(t) = 0$$

$$-1 < t < 0 \Rightarrow y(t) = t + 1$$

$$0 < t < 5 \Rightarrow y(t) = 1$$

$$5 < t < 6 \Rightarrow y(t) = -t + 6$$

$$t > 6 \Rightarrow y(t) = 0$$

